INTERVAL ESTIMATION – ONE SAMPLE FORMULAS

1 -Confidence Intervals

Interval estimation of a population mean $\mu$ when $\sigma$ is known

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

where $z_{\frac{\alpha}{2}}$ is the $\frac{\alpha}{2}$ critical value for a z-distribution. The confidence interval is exact for normal populations and is approximately correct for large samples of non normal populations.

Interval estimation of a population mean $\mu$ when $\sigma$ is unknown

$$\bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where $t_{\frac{\alpha}{2}}$ is the $\frac{\alpha}{2}$ critical value for a t-distribution with $n-1$ degrees of freedom. The confidence interval is exact for normal populations and is approximately correct for large samples of non normal populations.

Interval estimation of a population proportion

$$\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $\hat{p} = \frac{x}{n}$ is the sample proportion and $\hat{q} = 1 - \hat{p}$. Assumption: the number of successes, $x$, and the number of failures, $n-x$, are both greater than 5.

Interval estimation of a population variance

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

where $\chi^2_L$ and $\chi^2_R$ denote respectively the left and right-tailed critical values of the $\chi^2$ distribution with $n-1$ degrees of freedom. The confidence interval is exact for normal populations and is approximately correct for large samples of non normal populations.
2. Minimum Sample size for margin of error E

**Minimum sample size for estimating a population mean \( \mu \)**

\[
n = \left( \frac{z_{\alpha} \cdot \sigma}{E} \right)^2
\]

(Rounded up to the nearest whole number)

**Minimum sample size for estimating a population proportion \( p \)**

\[
n = \frac{z_{\alpha}^2 \cdot (.25)}{E^2}
\]

(Rounded up to the nearest whole number)

Used when there is no idea about \( p \) magnitude

\[
n = \frac{z_{\alpha}^2 \cdot p_g \cdot q_g}{E^2}
\]

(Rounded up to the nearest whole number)

where the "g" means an "educated guess" about the values