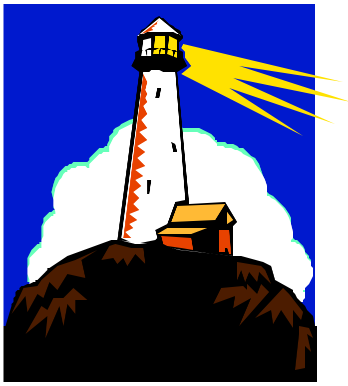

LOGIC – REVIEW

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STATEMENTS

A statement is a declarative sentence that is either **TRUE** or **FALSE**.

The truthfulness or falsity of a statement is called its ***truth value***.

Examples of statements:

Bayside is closed after 10:00 pm.

- Every multiple of 10 is a multiple of 5.
- Georgia is a southern state.

Examples of sentences that are not statements:

- Mathematics is a hard subject. (Opinion)
- Women are oppressed. (It may be true or false depending on other factors.)
- Study hard! (Imperative sentence)

Lower case letters symbolize statements:

p: "Every multiple of 5 is a multiple of 10."

q: "Georgia is a southern state."

r: "He studies every night."

s: "She is not used to travel so much."

COMPOUND STATEMENTS

Sentences that combine several statements are called **compound statements**. Later on there will be a discussion about connectives.

Examples of a compound statement:

- She is expecting a baby ***and*** she reads a book every week. (The connective in this compound statement is "***and***".)

- Charles speaks English well ***or*** he has forgotten his child years. (The connective in this compound statement is "***or***".)

CONJUNCTION

Connective: “**and**”

p Ù q : "p and q"

Example:

He is a veteran of the war ***and*** he is an ocean engineer.

<u>Truth Table for Conjunction</u>

p	**q**	**p Ù q**
T	T	T
T	F	F
F	T	F
F	F	F

Notice that a conjunction is TRUE only when both statements are TRUE!

DISJUNCTION

Connective: **“or”**

p Ú q : "p or q"

Example:

He is a veteran of war ***or*** he has a high regard for the love of his country.

Truth Table for Disjunction

p	**q**	**p Ú q**
T	T	T
T	F	T
F	T	T
F	F	F

Notice that a disjunction is FALSE only when both statements are FALSE!

NEGATION

$\sim p$: "not p"

Examples:

- Los Angeles is not in North Carolina.
- Tigers are not feline.

Truth Table for Negation

p	~ p
T	F
F	T

Notice that the truth value of a statement is the opposite of the truth value of its negation!

PROPOSITIONS

A compound statement in which the individual statements are expressed by letters without specifying their truth value is called a **proposition.**

Example of a proposition:

~ (p Ù ~ q): "It is not the case that p and not q"

(r Ú ~q): r or not q

The truth value of a proposition depends on the truth values of the involved statements. Propositions can be visualized in a concise way by using a truth table.

TRUTH TABLE

- In a truth table the first columns shows all the possible truth values for the composing statements. There should be enough rows to consider all the possible combination of truth values.
- Each of the remaining columns corresponds to a step in the construction of the proposition.

Example of a Truth Table:

Proposition: ~ (p Ù ~ q)

p	q	~	(p	Ù	~	q)
T	T	**T**	T	F	F	T
T	F	**F**	T	T	T	F
F	T	**T**	F	F	F	T
F	F	**T**	F	F	T	F

Truth value of the proposition

Another example of a Truth Table:

Proposition: ~ (p Ú ~ q)

p	q	~	(p	Ú	~	q)
T	T	**F**	T	T	F	T
T	F	**F**	T	T	T	F
F	T	**T**	F	F	F	T
F	F	**F**	F	T	T	F

Truth value of the proposition

One more example of a Truth Table:

Proposition: ~ (p Ù ~ q) Ú p

p	q	~	(p	Ù	~	q)	Ú	p
T	T	T	T	F	F	T	**T**	T
T	F	F	T	T	T	F	**T**	T
F	T	T	F	F	F	T	**T**	F
F	F	T	F	F	T	F	**T**	F

Truth value of the proposition

TAUTOLOGY

A proposition that is true independently of the truth value of the component statements.

In a tautology the final (resulting) column of the truth table consists of all T's.

Example of Tautology:

Proposition: p Ú ~ p

p	p	Ú	~	p
T	T	**T**	F	T
T	T	**T**	F	T
F	F	**T**	T	F
F	F	**T**	T	F

CONTRADICTION

A proposition that is false independently of the truth value of the component statements.

In a contradiction the final (resulting) column of the truth table consists of all F's.

Example of Contradiction:

Proposition: p Ù ~ p

p	p	Ù	~	p
T	T	**F**	F	T
T	T	**F**	F	T
F	F	**F**	T	F
F	F	**F**	T	F

LOGICAL EQUIVALENCE

Two propositions are logically equivalent when they have identical truth tables.

Logical Equivalence is denoted by the symbol ≡ .

Example:

Proposition: p º ~ (~ p)

Example of two Logically Equivalent propositions:

Proposition: ~ (p Ù q) º ~ p Ú ~ q

p	q	~	(p	Ù	q)
T	T	**F**	T	T	T
T	F	**T**	T	F	F
F	T	**T**	F	F	T
F	F	**T**	F	F	F

p	q	~	p	Ú	~	q
T	T	F	T	**F**	F	T
T	F	F	T	**T**	T	F
F	T	T	F	**T**	F	T
F	F	T	F	**T**	T	F

Same truth table

CONDITIONAL

If p, then q

p ® q

The conditional is frequently read as

"p implies q" or "p only if q"

p is called the **antecedent** and q is called the **consequent.**

Other expressions for $p \rightarrow q$ are:

"p is sufficient for q"
"q is necessary for p"

Example:

- If one sleeps during the flight, then the trip will seem short.
- Sleeping during the flight implies that the trip will seem short.
- Sleep during the flight only if the trip seems short.
- Sleeping during the flight is sufficient for the flight to seem shorter.
- The flight seems shorter is necessary for sleeping during it.

Truth Table for Conditional

p ® q

p	q	p ® q
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

Example of the four cases:

- If 4 > 3, then cats have four legs. (True)
- If 4 > 3, then cats do not have four legs. (False)
- If 4 < 3, then cats have four legs. (True)
- If 4 < 3, then cats do not have four legs. (True)

Notice that a conditional is false only when the antecedent is true and the consequent is false.

Note: This example should convince you that true and false statements in propositional logic sometimes do not "sound" like the way we usually talk.

BICONDITIONAL

p if and only if q

Biconditional is equivalent to “p ↔ q and q ↔ p”

p « q

Example:

A number is even if and only if it is divisible by 2.

<u>The Truth Table for Biconditional</u>

p	q	p « q
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

Example of the four cases:

- **4 > 3 if and only if cats have four legs. (True)**
- **4 > 3 if and only if cats do not have four legs. (False)**
- **4 < 3 if and only if cats have four legs. (False)**
- **4 < 3 if and only if cats do not have four legs. (True)**

Notice that a biconditional is false only when one of the components is false.
Notice also that this time the truth values of the statements are easier to digest.

MORE about CONDITIONALS

It can be shown that

~ (p ® **q)** º **p** Ù **~q** (both propositions are logically equivalent)

Thus we can say the negation of a conditional is the conjunction of the antecedent with the negation of the consequent.

Notice that both propositions have the same truth values.

p	q	~	(p	®	q)
T	T	**F**	T	T	T
T	F	**T**	T	F	F
F	T	**F**	F	T	T
F	F	**F**	F	T	F

p	q	p	Ù	~	q
T	T	T	**F**	F	T
T	F	T	**T**	T	F
F	T	F	**F**	F	T
F	F	F	**F**	T	F

Same truth values

Example:

The **negation** of the statement

> *"If I watch too much TV, then I turn myself into a couch potato."*

is **equivalent** to the statement

> *"I watch too much TV and I do not turn myself into a coach potato."*

It can be shown that

p ® **q** º **~p** Ú **q** (both propositions are logically equivalent)

Thus we can say that a conditional is equivalent to the disjunction of the negation of the antecedent with the consequent.

Notice that both have the same truth values.

p	**q**	**p** ® **q**
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

p	**q**	**~**	**p**	Ú	**q**
T	T	F	T	**T**	T
T	F	F	T	**F**	F
F	T	T	F	**T**	T
F	F	T	F	**T**	F

Same truth values

Example:

The statement

> *"If I watch too much TV, then I turn myself into a couch potato."*

is **equivalent** to the statement

> *"I do not watch too much TV or I turn myself into a couch potato."*

FOUR RELATED CONDITIONALS:

Direct: p ® q
Converse: q ® p
Inverse: ~ p ® ~ q
Contrapositive: ~ q ® ~ p

Example:

Direct:	"If JP is right, then we are late."
Converse:	"If we are late, then JP is right."
Inverse:	"If JP is not right, then we are not late."
Contrapositive:	"If we are not late, then JP is not right."

Property:

The Direct conditional is equivalent to the Contrapositive conditional.
The Converse conditional is equivalent to the Inverse conditional.

Examples:

To say:	If I love you, then I will commit myself to you for life. (Direct)
Is equivalent to say:	If I do not commit myself to you for life, then I do not love you. (Contrapositive)

To say:	If I commit myself to you for life, then I love you. (Converse)
Is equivalent to say:	If I do not love you, then I do not commit myself to you for life. (Inverse)

ARGUMENTS and LOGICAL IMPLICATIONS

An argument consists of two or more statements called “premises” and a statement that follows from the truth of the premises. This last statement is called the “conclusion”.

Example:

p Ù **q**
p

---

q

Premises: “p and q”
“p”
Conclusion: “q”

More examples of arguments:

All bananas are vegetables.
All vegetables are healthy foods.

All bananas are healthy foods.

If you love me, then I am happy.
You do not love me.

I am not happy.

ANALYZING ARGUMENTS

To analyze an argument we must construct the argument truth table of:

PREMISES $\rightarrow$ CONCLUSION

If we obtain T for **<u>all</u>** possible combinations of truth-values of the statements in the premises we have a **VALID ARGUMENT** or **TAUTOLOGY**. Otherwise, the argument is Invalid.

Example of argument analysis:

p Ù q	**(premise 1)**
p	**(premise 2)**
q	**(conclusion)**

Written as conditional: $[(p \wedge q) \wedge p] \rightarrow q$ (VALID)

p	q	[(p Ù q)	Ù	p]	®	q
T	T	T	T	T	**T**	T
T	F	F	F	T	**T**	F
F	T	F	F	F	**T**	T
F	F	F	F	F	**T**	F

Valid argument

Another example of argument analysis:

All bananas are vegetables.
All vegetables are healthy foods.

All bananas are healthy foods.

p: banana q: vegetable r: healthy food

$$p \to q$$
$$q \to r$$

$$p \to r$$

Argument: $[(p \to q) \wedge (q \to r)] \to (p \to r)$ (VALID)

p	**q**	**r**	**[(p® q)**	**Ù**	**(q ® r)]**	**®**	**p ® r**
T	T	T	T	T	T	**T**	T
T	T	F	T	F	F	**T**	F
T	F	T	F	F	T	**T**	T
T	F	F	F	F	T	**T**	F
F	T	T	T	T	T	**T**	T
F	T	F	T	F	F	**T**	T
F	F	T	T	T	T	**T**	T
F	F	F	T	T	T	**T**	T

T for all cases

Another example of argument analysis:

If you love me, then I am happy. (p → q)
You do not love me. (~p)

I am not happy. (~q)

Argument: [(p → q) Ù (~p)] → (~q)

p	q	[(p® q)	Ù	~p)]	®	~q
T	T	T	F	F	**T**	F
T	F	F	F	F	**T**	T
F	T	T	T	T	**F**	F
F	F	T	T	T	**T**	T

Invalid argument

There are some typical valid arguments (syllogisms) whose forms are worthy to memorize:

Modus Ponens	**Modus Tollens**	**Disjoint Syllogism**	**Reasoning by Transitivity**
p → q p ——— q	p → q ~ q ——— ~ p	p ∨ q ~ p ——— q	p → q q → r ——— p → r

Previously we analyzed a case of Reasoning by Transitivity. Go ahead and create examples to analyze the other three forms!

INVALID ARGUMENT FORMS (FALLACIES)

Fallacy of the Converse	Fallacy of the Inverse
$p \to q$ q ——— p	$p \to q$ $\sim p$ ——— $\sim q$

Complete the truth tables and you will see that both are Invalid arguments.

Fallacy of the Converse

Argument: [(p ® q) Ù q] ® p

p	q	[(p ® q)	Ù	q]	®	p
T	T	T	T	T	**T**	T
T	F	F	F	F	**T**	T
F	T	T	T	T	**F**	F
F	F	T	F	F	**T**	F

Examples of this fallacy:

- If a person is right, he/she will maintain his/her assertion. Martha firmly maintained her assertion. Therefore, Mary is right.
- If a number is rational, then it is a real number. Pi (Π) is a real number. Therefore, Pi (Π) is rational.

Fallacy of the Inverse

Argument: [(p ® q) Ù ~p] ® ~q

p	q	[(p ® q)	Ù	~p]	®	~q
T	T	T	F	F	**T**	F
T	F	F	F	F	**T**	T
F	T	T	T	T	**F**	F
F	F	T	T	T	**T**	T

Examples of this fallacy:

- If Sandra buys a new car, then she will be happy. She did not by a new. Therefore, Sandra is not happy.
- Every negative number is an integer. The number 2 is not a negative number. Therefore, the number two is not an integer.

Summary of Truth Tables

$p \vee q$ *is False only when both statements are False*
$p \wedge q$ *is True only when both statements are True*
$p \rightarrow q$ *is False only when p is True and q is false*
$p \leftrightarrow q$ *is True only when p and q have the same value*

Important Equivalences

$\sim(\sim p) \equiv \quad p$
$\sim(p \vee q) \equiv \quad \sim p \wedge \sim q$
$\sim(p \wedge q) \equiv \quad \sim p \vee \sim q$
$\sim(p \rightarrow q) \equiv \quad p \wedge \sim q$
$p \rightarrow q \equiv \quad \sim p \vee q$
$\sim p \rightarrow q \equiv \quad p \vee q$
$p \rightarrow q \equiv \quad \sim q \rightarrow \sim p$

Derived Conditionals

$p \rightarrow q$ *Direct*
$q \rightarrow p$ *Converse*
$\sim p \rightarrow \sim q$ *Inverse*
$\sim q \rightarrow \sim p$ *Contra – Positive*
Direct $\equiv$ *Contra – Positive*
Inverse $\equiv$ *Comverse*

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