COUNTING FORMULAS- AN INTRODUCTION

Factorials

The product of the n first positive integers is called **"n factorial"** and denoted by **n!**

n!= 1.2.3.4….n

Example 3! = 1.2.3 = 6
4! = 1.2.3.4. = 24

By convention, we will agree that 0! = 1

Permutation rule:
The number of possible permutations (different orderings) of n objects is n!

Example: If we have three objects A, B, and C.
The different permutations are:

ABC, ACB, BAC, BCA, CAB, CBA.

Thus we have 6 possible permutations. Notice that 3! = 6

Example: Mary has 4 children. She wants to put a picture of each one in the four pages of her wallet. In how many different ways can she put the pictures in her wallet?

Number of different orders = 4! = 1.2.3.4 = 24

Permutations

Permutation of **r** objects means the number of different ways in which the objects can be sorted. From a previous formula, we now that that is r!
Suppose that n objects ($n \geq r$) are given. Permutation of n objects, taken r at a time is:

$$P_r^n = \frac{n!}{(n-r)!}$$

Example: suppose that 4 objects, A, B, C, and D are given and we need the permutations of 2 of them. The permutations are:

AB, BA, AC, CA, AD, DA, BD, DB, CD, DC, BC, CB

By the above formula, we get:

$$P_2^4 = \frac{4!}{(4-2)!} = \frac{1.2.3.4.}{1.2} = 12$$

Example: A teacher has 5 students. She wants to select three students and engage the group in a joint project. Each student in the group will have a different task. In how many ways can she select the groups?

$$P_3^5 = \frac{5!}{(5-3)!} = \frac{1.2.3.4.5}{1.2} = 60$$

Notice that if one of the selections is John, Troy and Tricia, the different ways in which the tasks may be assigned to each of them generate 6 (of 3!) permutations. In other words permutations count not only for the triplet selected but also for the order of the objects in the same selection.

One permutation is for example:
John: Task 1
Troy: Task 2
Tricia: Task3
Other different permutation is:
Troy: Task 1
John: Task 2
Tricia: Task 3

Remember: Permutations of r elements selected from n element are arrays in which the order matters!

Combinations

Given n objects, we are interested in the different ways of selecting r of these objects ($n \geq r$). These are the combination of n objects, taken r at a time.

$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example:

A small college has a faculty of 10 professors. The Dean wants to select 4 professors for a task force meant to suggest some activities to enhance student responsibility. How many possible different groups of 4 professors can be selected from the 10 professors in the faculty?

Given objects: 10
Select groups of: 4
Order: not important

$$C_4^{10} = \frac{10!}{4!(10-4)!} = \frac{1.2.3.4.5.6.7.8.9.10}{1.2.3.4.1.2.3.4.5.6.} = 210$$

Consider this alternative situation. The dean is going to assign specific tasks to each faculty in the group of 4. One will survey students, other will study previous experiences in the topic, another will write the report and the fourth one will make an oral presentation to the dean and rest of the faculty. What are now the different possible selections?
Since now the order matters, the number of ways is given by the permutation of 10 taken 4 at a time

$$P_4^{10} = \frac{10!}{(10-4)!} = \frac{1.2.3.4.5.6.7.8.9.10}{1.2.3.4.5.6} = 504$$

Notice that the number of permutation is greater than the number of combination!

Multiplicative Rule

If we have two groups of objects, one with m_1 objects and the other with m_2 objects, the number of ways in which we can select one object of each group is

$$m_1 \cdot m_2 .$$

Example: If you have two shelves in your bookcase. One has 10 non-fiction books and the other contains 12 novels. There are 10x12 or 120 ways of selecting one non-fiction book and one novel.

The multiplicative rule can be generalized as follows. Given k groups of $m_i, i = 1,2,...k$ objects each, the number of ways of selecting one object from each book is given by

$$m_1 \cdot m_2 .m_3 ....m_k$$

Example. In a certain state license plates consist of three letters followed by three digits.
If there is no other restriction, the number of possible different license plates will be

26x26x26x10x10x10= 17,576,000

But if no letter or digit could be repeated the total number of possible different plates is

26x25x24x10x9x8=11,232,000

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Summary of Formulas

	For k groups with $m_1, m_2, ..., m_k$ objects respectively in each group, the multiplicative formula is: $m_1.m_2....m_k$ It gives the number of ways of selecting k objects, one object from each group
	Permutation of n elements taken r at a time $P_r^n = \frac{n!}{(n-r)!}$
	Combination of n elements taken r at a time $C_r^n = \frac{n!}{r!(n-r)!}$

Illustration

Consider a deck of 52 cards. Remember that there is 4 suits of 13 card in each deck. Two suits consist of red cards and the other two suits are black cards. Each suit has 3 face cards

a) How many 5-card hands can be selected from the deck?

Number of 5 card hands from a deck of 52 cards: $C_5^{52} = \frac{52!}{5!(52-5)!} = 2{,}598{,}960$

b) In how many ways can we select four cards en such a way that no suit is repeated?

By multiplicative formula this is 13X13x13x13 = 28,561

c) In a certain card game each of 5 gamblers receive one card. The one with highest card value will distribute the cards in the first game run. In how many ways the 5 cards from the deck will be distributed among the 5 gamblers?

$$P_5^{52} = \frac{52!}{5!(52-5)!)} = 1{,}299{,}480$$

d) In how many ways can we select a hand of 3 red cards and 2 black cards:

$$C_3^{26} C_2^{26} = \frac{26!}{3!(26-3)!} * \frac{26!}{2!(26-2)!} = 2{,}600 * 325 = 845{,}000$$

e) In how many ways can we select a 5-card hand with exactly 2 face cards?

$$C_2^{12} C_3^{40} = \frac{12!}{2!(12-2)!} * \frac{40!}{3!(40-3)!} = 66 * 9{,}880 = 652{,}080$$